1 Introduction

The focus of my research lies in an intersection of noncommutative representation theory, Lie theory, and invariant theory. I study algebra-subalgebra pairs that form so-called semicommutative pairs $\Gamma \subset \mathscr{U}$, where $\mathscr{U}$ is an associative (noncommutative) $\mathbb{C}$-algebra and Γ is an integral domain [LM73; Žel73; DFO94; FO10]. This situation generalizes the pair $\Gamma \subset U(\mathfrak{gl}_n)$ where Γ is the *Gelfand-Tsetlin* subalgebra $\Gamma = \mathbb{C}\langle \cup_{k=1}^{n} Z(U(\mathfrak{gl}_k))\rangle$ [GT50; DFO94]. Historically, this began with the work of Gelfand and Tsetlin with their foundational paper [GT50] in which they showed that finite-dimensional irreducible $U(\mathfrak{gl}_n)$-modules have a basis parameterized by Gelfand-Tsetlin patterns.

Definition 1.1. A *Gelfand-Tsetlin pattern* is a tableau of $\lambda_{ij} \in \mathbb{C}$ for $1 \leq j \leq i \leq n$ arranged as follows:

$$\begin{array}{ccccccc}
\boxed{\lambda_{n1}} & \boxed{\lambda_{n2}} & & \cdots & & \boxed{\lambda_{n,n-1}} & \boxed{\lambda_{nn}} \\
& \boxed{\lambda_{n-1,1}} & & \cdots & & \boxed{\lambda_{n-1,n-1}} & \\
& & \ddots & & \iddots & & \\
& & \boxed{\lambda_{21}} & & \boxed{\lambda_{22}} & & \\
& & & \boxed{\lambda_{11}} & & &
\end{array}$$

Where the λ_{ki} are subject to the following interleaving relations:

(1) $\lambda_{k,i} - \lambda_{k-1,i} \in \mathbb{Z}_{\geq 0}$, and

(2) $\lambda_{k-1,i} - \lambda_{k,i+1} \in \mathbb{Z}_{\geq 0}$.

$U(\mathfrak{gl}_n)$ acts on these patterns by rational functions in the entries and integer shifts of the entries [GT50; DFO94]. By construction, Γ acts diagonally in these bases. Another remarkable property of Γ is that $U(\mathfrak{gl}_n)$ is free as a left Γ-modules [Ovs03]. In 2010, Futorny and Ovsienko showed that there is an embedding of $U(\mathfrak{gl}_n)$ into a ring of invariants in a skew group algebra. One reason we study the representations of $U(\mathfrak{gl}_n)$ is that they are equivalent to representations of $\mathfrak{gl}_n$, but $U(\mathfrak{gl}_n)$ is an associative algebra and therefore "easier" to study than $\mathfrak{gl}_n$ even though it is a larger algebra. The results of Futorny and Ovsienko [FO10] connect $U(\mathfrak{gl}_n)$ representations to a still larger algebra that again makes it "easier" to study its representations.

Based on these observations, in [FO10], Futorny and Ovsienko defined *Galois rings* and *Galois orders* (see Definitions 1.3 and 1.4 respectively), and studied their representations, and in [Har20] Hartwig introduced a more streamlined approach to describe these objects. To describe them, we need the following data: $(\Lambda, G, \mathscr{M})$ where Λ is an integrally closed domain, G is a finite subgroup of $\mathrm{Aut}(\Lambda)$, and $\mathscr{M}$ is a submonoid of $\mathrm{Aut}(\Lambda)$. Additionally, this data adheres to the following assumptions from [Har20]:

(1) $(\mathscr{M}\mathscr{M}^{-1}) \cap G = 1_{\mathrm{Aut}(\Lambda)}$,

(2) $\forall g \in G, \forall \mu \in \mathscr{M} : {}^{g}\mu = g \circ \mu \circ g^{-1} \in \mathscr{M}$,

(3) Λ is Noetherian as a module over Λ^G.

By Assumption (2) and that G acts naturally on $\mathrm{Frac}(\Lambda)$ (due to being a subgroup of $\mathrm{Aut}(\Lambda)$), G acts on $\mathrm{Frac}(\Lambda)\#\mathscr{M}$, *the skew monoid ring*, which is defined as the free left $\mathrm{Frac}(\Lambda)$-module on $\mathscr{M}$ with multiplication give by $a_1\mu_1 \cdot a_2\mu_2 = (a_1\mu_1(a_2))(\mu_1\mu_2)$ for $a_i \in \mathrm{Frac}(\Lambda)$ and $\mu_i \in \mathscr{M}$. In this setting, we define $\Gamma := \Lambda^G$, the subring G invariant elements of Λ.

Definition 1.2 ([Har20])**.** For any element $X \in \operatorname{Frac}(\Lambda)\#\mathscr{M}$, we define a $\mathbb{Z}$-bilinear map from $(\operatorname{Frac}(\Lambda)\#\mathscr{M})\times \operatorname{Frac}(\Lambda) \to \operatorname{Frac}(\Lambda)$ called the *evaluation of X at f* for an element $f \in \operatorname{Frac}(\Lambda)$ by:

$$X(f) = \sum_{\mu \in \mathscr{M}} a_\mu \cdot \mu(f).$$

Now we can define the objects of interest in my research.

Definition 1.3 ([FO10])**.** A finitely generated Γ-subring $\mathscr{U} \subseteq (\operatorname{Frac}(\Lambda)\#\mathscr{M})^G$ is called a *Galois* Γ*-ring* (or *Galois ring with respect to* Γ) if $\operatorname{Frac}(\Gamma)\mathscr{U} = \mathscr{U}\operatorname{Frac}(\Gamma) = (\operatorname{Frac}(\Lambda)\#\mathscr{M})^G$.

In other words, if we localize Γ inside of $\mathscr{U}$, we attain all of the G invariant elements of $\operatorname{Frac}(\Lambda)\#\mathscr{M}$.

Definition 1.4 ([FO10])**.** A Galois Γ-ring $\mathscr{U}$ in $\mathscr{K}$ is a *left* (respectively *right*) *Galois* Γ*-order in* $\mathscr{K}$ if for any finite-dimensional left (respectively right) K-subspace $W \subseteq \mathscr{K}$, $W \cap \mathscr{U}$ is a finitely generated left (respectively right) Γ-module. A Galois Γ-ring $\mathscr{U}$ in $\mathscr{K}$ is a *Galois* Γ*-order in* $\mathscr{K}$ if $\mathscr{U}$ is a left and right Galois Γ-order in $\mathscr{K}$.

The condition in this original definition is very technical and difficult to show in general. In 2017, Hartwig showed that the following condition implies the original.

Definition 1.5. Let $\mathscr{U}$ be a Galois Γ-ring such that $X(\Gamma) \subseteq \Gamma$ for every $X \in \mathscr{U}$. Then $\mathscr{U}$ is Galois Γ-order called a *principal Galois* Γ*-order*.

Galois orders form a collection of algebras that contains many important examples, including: *generalized Weyl algebras* defined by independently by Bavula [Bav92] and Rosenberg [Ros95] in the early nineties, the universal enveloping algebra of $\mathfrak{gl}_n$, shifted Yangians and finite W-algebras [FMO10], Coulomb branches [Web19], and $U_q(\mathfrak{gl}_n)$ [FH14; Har20]. Their structures and representations have been studied in [Fut+18], [FS18b], [Har20], [MV18], and [Web19].

Defining these objects unifies the representation theory of these algebras. In particular, unifying the study of *Gelfand-Tsetlin* modules.

Definition 1.6. A $\mathscr{U}$-module V is a *Gelfand-Tsetlin* module (with respect to Γ) if $\dim(\Gamma.v) < \infty$ for all $v \in V$.

The major results in [FO14] give us the following:

(1) The existence of "generic" simple Gelfand-Tsetlin modules over Galois rings.

(2) A "rough" classification of simple Gelfand-Tsetlin modules over Galois orders.

Moreover for principal Galois orders, Hartwig has the following result:

Theorem 1.7 ([Har20], Theorem 3.3 (i))**.** *Let $\xi \in \hat{\Gamma}$ be any character. If $\mathscr{U}$ is a principal Galois Γ-order in $(\operatorname{Frac}(\Lambda)\#\mathscr{M})^G$, then the right cyclic $\mathscr{U}$-module $\xi\mathscr{U}$ has a unique simple quotient $V(\xi)$. Moreover, $V(\xi)$ a Gelfand-Tsetlin over $\mathscr{U}$ with $V(\xi)_\xi \neq 0$ and is called the canonical simple left Gelfand-Tsetlin $\mathscr{U}$-module associated to ξ.*

The current research in this area has been focused on principal Galois orders, as they contain all of the examples of interest, and the condition that $X(\Gamma) \subseteq \Gamma$ is much easier to verify. In particular in [Web19], Webster showed that some principal Galois orders are Morita equivalent to a *principal flag order* which is a Galois order in which the G is trivial and $\mathscr{M}$ is the semidirect product of the group and monoid from the original data (see Lemma 2.5 in [Web19]). Additionally, in [LW19] it is shown that spherical Cherednik algebras are principal Galois orders.

Definition 1.8. A *principal flag order* with data $(\Lambda, G \ltimes \mathscr{M})$ is a subalgebra of $F \subset \mathrm{Frac}(\Lambda)\#(G \ltimes \mathscr{M})$ such that:

(i) $\Lambda \# G \subset F$,

(ii) $\mathrm{Frac}(\Lambda)F = \mathrm{Frac}(\Lambda)\#(G \ltimes \mathscr{M})$,

(iii) For every $X \in F$, $X(\Lambda) \subset \Lambda$.

Definition 1.9. The *standard flag order* with data $(\Lambda, G \ltimes \mathscr{M})$ is the subalgebra of all elements X in $F \subset \mathrm{Frac}(\Lambda)\#(G \ltimes \mathscr{M})$ satisfying (iii) and is denoted $\mathcal{F}_\Lambda$.

The current work with these algebras shows that they generalize nilHecke algebras.

2 Completed work

In [FO10], Futorny and Ovsienko described $U(\mathfrak{gl}_n)$ as the subalgebra of the ring of invariants of a certain noncommutative ring with respect to the action of $S_1 \times S_2 \times \cdots \times S_n$ where S_j is the symmetric group on j variables such that $U(\mathfrak{gl}_n)$ was a Galois order with respect to its Gelfand-Tsetlin subalgebra Γ (Denoted U_n).

We recall in Galois theory, given a Galois extension L/K with $\mathrm{Gal}(L/K) = G$ the subgroups $\widetilde{G}$ of G correspond to intermediate fields $\widetilde{K}$ with $\mathrm{Gal}(L/\widetilde{K}) = \widetilde{G}$ with normal subgroups of particular interest. Since S_n has only one normal subgroup for $n \geq 5$, one might wonder what the object similar to $U(\mathfrak{gl}_n)$ would be if we considered the invariants with respect to the normal subgroup $\mathbb{A}_n := A_1 \times A_2 \times \cdots \times A_n$ where A_j is the alternating group on j variables. My publication, in the Journal of Algebra [Jau21], describes such an algebra defined below:

Definition 2.1 ([Jau21])**.** The following extension of $U(\mathfrak{gl}_n)$, denoted $\mathscr{A}(\mathfrak{gl}_n)$, defined as the subalgebra of $(\mathbb{C}(x_{ki} \mid 1 \leq i \leq k \leq n)\#\langle \delta^{\ell j} \mid 1 \leq j \leq \ell \leq n-1\rangle_{\mathrm{grp}})^{\mathbb{A}_n}$ generated by $U_n \cup \{\mathcal{V}_2, \mathcal{V}_3, \cdots, \mathcal{V}_n\}$. Explicitly, $\mathscr{A}(\mathfrak{gl}_n)$ is the subalgebra generated by

$$\begin{array}{ll} X_k^{\pm} = \sum_{i=1}^{k} (\delta^{ki})^{\pm 1} a_{ki}^{\pm} & \text{for } k = 1, \ldots, n-1, \\ X_{kk} = \sum_{j=1}^{k} (x_{kj} + j - 1) - \sum_{i=1}^{k-1} (x_{k-1,i} + i - 1) & \text{for } k = 1, \ldots, n, \\ \mathcal{V}_k = \mathcal{V}_k(x_{k1}, \ldots, x_{kk}) = \prod_{i<j} (x_{ki} - x_{kj}) & \text{for } k = 1, \ldots, n-1, \end{array}$$

where $a_{ki}^{\pm}$ are described by Futorny and Ovsienko in [FO10].

This provides the first natural example of a Galois ring whose ring Γ is not a semi-Laurent polynomial ring, that is, a tensor product of polynomial rings and Laurent polynomial rings. Additionally, our symmetry group $A_1 \times A_2 \times \cdots \times A_n$ is not a complex reflection group while the research in this area has been almost exclusively focused on complex reflection groups. The algebra $\mathscr{A}(\mathfrak{gl}_n)$ is an extension of $U(\mathfrak{gl}_n)$ by $n-1$ elements $\mathcal{V}_2, \ldots, \mathcal{V}_n$. I prove some properties of $\mathscr{A}(\mathfrak{gl}_n)$ that are quite similar to $U(\mathfrak{gl}_n)$:

Proposition 2.2 ([Jau21], Proposition 2.2)**.**

(i) $U(\mathfrak{gl}_n) \cong U_n \subset \mathscr{A}(\mathfrak{gl}_n)$,

(ii) $\mathscr{A}(\mathfrak{gl}_n)$ *is a Galois* $\widetilde{\Gamma}$*-ring,*

(iii) $\mathcal{V}_n$ *is central in* $\mathscr{A}(\mathfrak{gl}_n)$,

(iv) $Z(\mathscr{A}(\mathfrak{gl}_n)) \cong \mathbb{C}[x_1, \ldots, x_n]^{A_n}$,

(v) *there is a chain of subalgebras* $\mathscr{A}(\mathfrak{gl}_1) \subset \mathscr{A}(\mathfrak{gl}_2) \subset \cdots \subset \mathscr{A}(\mathfrak{gl}_n)$,

(vi) $\mathscr{A}(\mathfrak{gl}_n)$ *is a minimal extension of* $U(\mathfrak{gl}_n)$ *with properties* (iv) *and* (v).

Property (iv) shows that the "Weyl Group" of $\mathscr{A}(\mathfrak{gl}_n)$ is the alternating group A_n, in the sense that there is a natural extension $\widetilde{\varphi}_{\mathrm{HC}}$ of the Harish-Chandra homomorphism $\varphi_{\mathrm{HC}}\colon Z(U(\mathfrak{gl}_n)) \to S(\mathfrak{h}) \cong \mathbb{C}[x_1, \dots, x_n]$, such that

$$\widetilde{\varphi}_{\mathrm{HC}}\colon Z(\mathscr{A}(\mathfrak{gl}_n)) \xrightarrow{\cong} \mathbb{C}[x_1, \dots, x_n]^{A_n}.$$

Additionally, I provide a technique to turn a general Galois ring into a Galois order that is related to localization (see Theorem 6.2 in [Jau21]). The thought of changing a Galois ring into a Galois order was something that had not been considered in the field before. This result provides some connections between three different principal Galois orders for a general $(\Lambda, G, \mathscr{M})$. There is the following chain of embeddings:

$$(\Lambda \# \mathscr{M})^G \hookrightarrow \mathscr{K}_\Gamma \hookrightarrow (\mathrm{Frac}(\Lambda)\#\mathscr{M})^G.$$

Hartwig showed that $(\Lambda\#\mathscr{M})^G$ is always a principal Galois Γ-order ([Har20] Lemma 2.10 and Lemma 2.16). He defined the *standard Galois* Γ*-order* denoted $\mathscr{K}_\Gamma$ ([Har20] Definition 2.22 and Theorem 2.21) which contains all principal Galois Γ-orders, and he showed that $(\mathrm{Frac}(\Lambda)\#\mathscr{M})^G$ is a (principal) Galois Γ-order iff Λ is a field (i.e. $\Lambda = \mathrm{Frac}(\Lambda)$). My technique in a sense brings the left and right ends closer together. Even if you cannot directly apply the technique, it will allow you to describe a new Galois order related to your original Galois ring to describe Gelfand-Tsetlin modules over the original Galois ring.

Finally, I compute the division ring of fractions and prove, for $n \leq 5$, $\mathscr{A}(\mathfrak{gl}_n)$ satisfies the Gelfand-Kirillov conjecture (see [GK66]). For the latter, I use Maeda's positive solution to Noether's problem for the alternating group A_5 [Mae89], and Futorny-Schwarz's Theorem 1.1 in [FS18a].

3 Ongoing work

3.1 Connecting related flag/Galois orders

As is often the case when defining a new class of objects, one of the first steps is to describe maps between these objects. My pre-print [Jau22] provides a sufficient condition to construct maps between flag orders and Galois orders. This allows us to study the similarities between orders with sufficiently related data.

Theorem 3.1 ([Jau22], Theorem 3.3)**.** *Let* $(\Lambda_i, \mathscr{M}_i, W_i)$ *for* $i = 1, 2$ *be the datum for standard flag orders* $\mathcal{F}_{\Lambda_i}$*. Let* $\Lambda = \Lambda_1 \otimes \Lambda_2$*,* $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$*, and* $W = W_1 \times W_2$*. There exist the following series of embeddings*

$$\mathcal{F}_{\Lambda_1} \otimes \mathcal{F}_{\Lambda_2} \hookrightarrow \mathcal{F}_\Lambda \hookrightarrow (\mathcal{F}_1 \otimes \mathcal{F}_2)_\Lambda.$$

Using this result, I proved that both principal flag orders and principal Galois orders are closed under tensor products. The construction of these maps and tensor products are the first steps in studying the category of flag/Galois orders.

3.2 Alternating nil Hecke algebras

In [Mit01], there is a description of a q-analogue of the alternating group, the alternating Hecke algebra, as a subalgebra of the Iwahori-Hecke algebra of the symmetric group. The current work in the field has demonstrated in this setting that the standard flag order associated to the data $(\mathbb{C}[x_1, \dots, x_n], S_n, 1)$ is isomorphic to the nil Hecke algebra (see [GKV97] and [Rou05]). In my pre-print [Jau22], I proved that the standard order for the data $(\mathbb{C}[x_1, \dots, x_n], A_n, 1)$ is a subalgebra of this nil Hecke algebra. I am currently working on using the alternating Hecke algebra of Mitsuhashi from [Mit01] to describe this subalgebra that I am calling the *nil Hecke algebra of the alternating group.*

4 Future work

In [Jau21], I have conjectured a presentation for $\mathscr{A}(\mathfrak{gl}_3)$ and I plan to show that it is a presentation soon. I have some thoughts on describing some infinite-dimensional weight modules for $\mathscr{A}(\mathfrak{gl}_n)$. In [Mat00], it is shown that any irreducible torsion-free infinite-dimensional weight module with finite-dimensional weight spaces is isomorphic to a "twisted localization" of $L(\lambda)$, the unique irreducible quotient of the Verma module associated to the weight λ. I hope to relate these types of modules for $\mathscr{A}(\mathfrak{gl}_n)$ to $U(\mathfrak{gl}_n)$-modules in the same way as I did for finite-dimensional modules (see Theorem 5.2 in [Jau21]). Some additional directions for $\mathscr{A}(\mathfrak{gl}_n)$ involve connecting it to the theory of Lie superalgebras, and Schur-Weyl duality for the alternating group.

In the future, I would also like to obtain a more natural description of the properties of Galois orders and relate them to algebraic geometry. Many properties that may seem perplexing when viewed through the lens of algebra, could be better understood geometrically.

Additionally, a problem that I am fascinated by is finding a Galois order realization of $U(\mathfrak{sp}_{2n})$. Doing so would prove the Gelfand-Kirillov Conjecture for $\mathfrak{sp}_{2n}$ which has been open for over 50 years.

5 Teaching and scholarship work

In addition to my mathematical research, I am beginning to work to delve into the world of research of teaching and scholarship. My desires for doing this are two-fold. Firstly, I have a passion for mathematical education, training future educators, and working with students transitioning to higher education. Secondly, this will allow me to work with more undergraduate students on research and share with them the joys of academia.

To this end, I am working with Dr. Katrina Rothrock (rothroks@uwec.edu) developing a program to be implemented at Stanley-Boyd High School to improve the mathematical readiness of college-bound seniors. One goal of this project is to counteract the standard remedial approach to mathematics. We are seeing these populations growing at an alarming rate and this often serves as a barrier to students successfully completing degree programs and disproportionately affects underrepresented students. We will be working closely with math teachers at Stanley-Boyd during intervention times to work on not just improving their mathematical ability, but other skills key to success in higher education, such as mindsets, self-efficacy, and self-regulated learning. We are applying for a National Council of Teachers of Mathematics (NCTM) MET grant to fund this program, which will serve as a pilot for future work with other local high schools. As part of this program, we will involve undergraduate math education majors to serve as "educational assistants" to help provide our students with more in-class experience. Additionally, we plan to involve them development and reporting process of the program.

I look forward to developing this area of research further in the future. Some directions I plan to investigate are effective teaching practices in introductory mathematics courses and how certain introductory mathematics courses form barriers to a student's ability to successfully complete their degree programs.

6 Undergraduate work

As I stated above, most of the work in this area has been focused in the setting where G is a complex reflection group. This leaves several accessible problems for small groups that are not complex reflection groups. I am currently working on a grant-funded project with an undergraduate to describe standard flag orders for some small non-complex reflection groups including the quaternion group and the alternating group. These would provide new and unique examples of flag orders in a field that has been dominated by complex reflection groups and would be analogs to the nil Hecke algebra of the symmetric group. We are planning to present the results of this project at the NCUR conferences at UWEC in the spring of 2023.

Additionally, I will work with undergraduates as part of the program described above with Dr. Rothrock. I hope to involve more undergraduates in future teaching and scholarship research. This area of my study will be approachable to more students and will allow students to be involved in research earlier in their academic careers.

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