

# 1 Introduction

The focus of my research has primarily been an intersection of noncommutative representation theory, Lie theory, and invariant theory. I study algebra-subalgebra pairs that form so called semicommutative pairs  $\Gamma \subset \mathcal{U}$ , where  $\mathcal{U}$  is an associative (noncommutative)  $\mathbb{C}$ -algebra and  $\Gamma$  is an integral domain [LM73; Žel73; DFO94; FO10]. This situation generalizes the pair  $\Gamma \subset U(\mathfrak{gl}_n)$  where  $\Gamma$  is the *Gelfand-Tsetlin* subalgebra  $\Gamma = \mathbb{C}\langle \cup_{k=1}^n Z(U(\mathfrak{gl}_k)) \rangle$  [GT50; DFO94]. Historically, this began with the work of Gelfand and Tsetlin with their foundational paper [GT50] in which they showed that finite-dimensional irreducible  $U(\mathfrak{gl}_n)$ -modules have a basis parameterized by Gelfand-Tsetlin patterns.

**Definition 1.1.** A *Gelfand-Tsetlin pattern* is a tableau of  $\lambda_{ij} \in \mathbb{C}$  for  $1 \leq j \leq i \leq n$  arranged as follows:

$$\begin{array}{ccccccc}
 \boxed{\lambda_{n1}} & \boxed{\lambda_{n2}} & & \cdots & & \boxed{\lambda_{n,n-1}} & \boxed{\lambda_{nn}} \\
 & \boxed{\lambda_{n-1,1}} & & \cdots & & \boxed{\lambda_{n-1,n-1}} & \\
 & & & \ddots & & & \ddots \\
 & & & & & \boxed{\lambda_{21}} & \boxed{\lambda_{22}} \\
 & & & & & \boxed{\lambda_{11}} & 
 \end{array}$$

Where the  $\lambda_{ki}$  are subject to the following interleaving relations:

- (1)  $\lambda_{k,i} - \lambda_{k-1,i} \in \mathbb{Z}_{\geq 0}$ , and
- (2)  $\lambda_{k-1,i} - \lambda_{k,i+1} \in \mathbb{Z}_{\geq 0}$ .

$U(\mathfrak{gl}_n)$  acts on these patterns by rational functions in the entries and integer shifts of the entries [GT50; DFO94]. By construction,  $\Gamma$  acts diagonally in these bases. Another remarkable property of  $\Gamma$  is that  $U(\mathfrak{gl}_n)$  is free as a left  $\Gamma$ -modules [Ovs03]. In 2010, Futorny and Ovsienko showed that there is an embedding of  $U(\mathfrak{gl}_n)$  into a ring of invariants in a skew group algebra. One reason we study the representations of  $U(\mathfrak{gl}_n)$  is they are equivalent to representations of  $\mathfrak{gl}_n$ , but  $U(\mathfrak{gl}_n)$  is an associative algebra and therefore “easier” to study than  $\mathfrak{gl}_n$  even though it is a larger algebra. The results of Futorny and Ovsienko [FO10] connect  $U(\mathfrak{gl}_n)$  representations to a still larger algebra that again makes it “easier” to study its representations.

Based on these observations, in [FO10], Futorny and Ovsienko defined *Galois rings* and *Galois orders* (see Definitions 1.3 and 1.4 respectively), and studied their representations, and in [Har20] Hartwig introduced a more streamlined approach to describe these objects. To describe them, we need the following data:  $(\Lambda, G, \mathcal{M})$  where  $\Lambda$  is an integrally closed domain,  $G$  is a finite subgroup of  $\text{Aut}(\Lambda)$ , and  $\mathcal{M}$  is a submonoid of  $\text{Aut}(\Lambda)$ . Additionally, this data adheres to the following assumptions from [Har20]:

- (1)  $(\mathcal{M}\mathcal{M}^{-1}) \cap G = 1_{\text{Aut}(\Lambda)}$ ,
- (2)  $\forall g \in G, \forall \mu \in \mathcal{M} : {}^g\mu = g \circ \mu \circ g^{-1} \in \mathcal{M}$ ,
- (3)  $\Lambda$  is Noetherian as a module over  $\Lambda^G$ .

As  $G$  is a subgroup of  $\text{Aut}(\Lambda)$  it naturally acts on  $\text{Frac}(\Lambda)$  and by Assumption (2); therefore,  $G$  acts on  $\text{Frac}(\Lambda)\#\mathcal{M}$ , the skew monoid ring, which is defined as the free left  $\text{Frac}(\Lambda)$ -module on  $\mathcal{M}$  with multiplication given by  $a_1\mu_1 \cdot a_2\mu_2 = (a_1\mu_1(a_2))(\mu_1\mu_2)$  for  $a_i \in \text{Frac}(\Lambda)$  and  $\mu_i \in \mathcal{M}$ . In this setting, we define  $\Gamma := \Lambda^G$ , the subring of  $G$ -invariant elements of  $\Lambda$ .

**Definition 1.2** ([Har20]). For any element  $X \in \text{Frac}(\Lambda)\#\mathcal{M}$  we define a  $\mathbb{Z}$ -bilinear map from  $(\text{Frac}(\Lambda)\#\mathcal{M}) \times \text{Frac}(\Lambda) \rightarrow \text{Frac}(\Lambda)$  called the *evaluation of  $X$  at  $f$*  for an element  $f \in \text{Frac}(\Lambda)$  by:

$$X(f) = \sum_{\mu \in \mathcal{M}} a_\mu \cdot \mu(f).$$

Now we can define the objects of interest in my research.

**Definition 1.3** ([FO10]). A finitely generated  $\Gamma$ -subring  $\mathcal{U} \subseteq (\text{Frac}(\Lambda)\#\mathcal{M})^G$  is called a *Galois  $\Gamma$ -ring* (or *Galois ring with respect to  $\Gamma$* ) if  $\text{Frac}(\Gamma)\mathcal{U} = \mathcal{U}\text{Frac}(\Gamma) = (\text{Frac}(\Lambda)\#\mathcal{M})^G$ .

In other words, if we localize  $\Gamma$  inside of  $\mathcal{U}$ , we attain all of the  $G$ -invariant elements of  $\text{Frac}(\Lambda)\#\mathcal{M}$ .

**Definition 1.4** ([FO10]). A Galois  $\Gamma$ -ring  $\mathcal{U}$  in  $\mathcal{K}$  is a *left* (respectively *right*) *Galois  $\Gamma$ -order in  $\mathcal{K}$*  if for any finite-dimensional left (respectively right)  $K$ -subspace  $W \subseteq \mathcal{K}$ ,  $W \cap \mathcal{U}$  is a finitely generated left (respectively right)  $\Gamma$ -module. A Galois  $\Gamma$ -ring  $\mathcal{U}$  in  $\mathcal{K}$  is a *Galois  $\Gamma$ -order in  $\mathcal{K}$*  if  $\mathcal{U}$  is a left and right Galois  $\Gamma$ -order in  $\mathcal{K}$ .

The condition in this original definition is very technical and difficult to show in general. In 2017, Hartwig showed that  $\mathcal{U}$  that the following condition implies the original.

**Definition 1.5.** Let  $\mathcal{U}$  be a Galois  $\Gamma$ -ring such that  $X(\Gamma) \subseteq \Gamma$  for every  $X \in \mathcal{U}$ . Then  $\mathcal{U}$  is Galois  $\Gamma$ -order called a *principal Galois  $\Gamma$ -order*.

Galois orders form a collection of algebras that contains many important examples, including: *generalized Weyl algebras* defined by independently by Bavula [Bav92] and Rosenberg [Ros95] in the early nineties, the universal enveloping algebra of  $\mathfrak{gl}_n$ , shifted Yangians and finite  $W$ -algebras [FMO10], Coulomb branches [Web19], and  $U_q(\mathfrak{gl}_n)$  [FH14; Har20]. Their structures and representations have been studied in [Fut+18], [FS18a], [Har20], [MV18], and [Web19].

Defining these objects unifies the representation theory of these algebras. In particular, unifying the study of *Gelfand-Tsetlin modules*.

**Definition 1.6.** A  $\mathcal{U}$ -module  $V$  is a *Gelfand-Tsetlin module* (with respect to  $\Gamma$ ) if  $\dim(\Gamma.v) < \infty$  for all  $v \in V$ .

The major results in [FO14] give us the following:

- (1) The existence of “generic” simple Gelfand-Tsetlin modules over Galois rings.
- (2) A “rough” classification of simple Gelfand-Tsetlin modules over Galois orders.

Moreover for principal Galois orders, Hartwig has the following result:

**Theorem 1.7** ([Har20], Theorem 3.3 (i)). *Let  $\xi \in \hat{\Gamma}$  be any character. If  $\mathcal{U}$  is a principal Galois  $\Gamma$ -order in  $(\text{Frac}(\Lambda)\#\mathcal{M})^G$ , then the right cyclic  $\mathcal{U}$ -module  $\xi\mathcal{U}$  has a unique simple quotient  $V(\xi)$ . Moreover,  $V(\xi)$  a Gelfand-Tsetlin over  $\mathcal{U}$  with  $V(\xi)_\xi \neq 0$  and is called the canonical simple left Gelfand-Tsetlin  $\mathcal{U}$ -module associated to  $\xi$ .*

The current research in this area has been focused on principal Galois orders, as they contain all of the examples of interest, and the condition that  $X(\Gamma) \subseteq \Gamma$  is much easier to verify. In particular in [Web19], Webster showed that some principal Galois orders are Morita equivalent to a *principal flag order* which is a Galois order in which the  $G$  is trivial and  $\mathcal{M}$  is the semidirect product of the group and monoid from the original data (see Lemma 2.5 in [Web19]). Additionally, in [LW19] it is shown that spherical Cherednik algebras are principal Galois orders.

**Definition 1.8.** A *principal flag order* with data  $(\Lambda, G \times \mathcal{M})$  is a subalgebra of  $F \subset \text{Frac}(\Lambda)\#(G \times \mathcal{M})$  such that:

- (i)  $\Lambda\#G \subset F$ ,
- (ii)  $\text{Frac}(\Lambda)F = \text{Frac}(\Lambda)\#(G \times \mathcal{M})$ ,
- (iii) For every  $X \in F$ ,  $X(\Lambda) \subset \Lambda$ .

**Definition 1.9.** The *standard flag order* with data  $(\Lambda, G \times \mathcal{M})$  is the subalgebra of all elements  $X$  in  $F \subset \text{Frac}(\Lambda)\#(G \times \mathcal{M})$  satisfying (iii) and is denoted  $\mathcal{F}_\Lambda$ .

The current work with these algebras shows that they generalize nilHecke algebras.

## 2 Completed work

In [FO10], Futorny and Ovsienko described  $U(\mathfrak{gl}_n)$  as the subalgebra of the ring of invariants of a certain noncommutative ring with respect to the action of  $S_1 \times S_2 \times \dots \times S_n$  where  $S_j$  is the symmetric group on  $j$  variables such that  $U(\mathfrak{gl}_n)$  was a Galois order with respect to its Gelfand-Tsetlin subalgebra  $\Gamma$ .

We recall in Galois theory, given a Galois extension  $L/K$  with  $\text{Gal}(L/K) = G$  the subgroups  $\tilde{G}$  of  $G$  correspond to intermediate fields  $\tilde{K}$  with  $\text{Gal}(L/\tilde{K}) = \tilde{G}$  with normal subgroups of particular interest. Since  $S_n$  has only one normal subgroup for  $n \geq 5$ , one might wonder what the object similar to  $U(\mathfrak{gl}_n)$  would be if we considered the invariants with respect to the normal subgroup  $\mathbb{A}_n := A_1 \times A_2 \times \dots \times A_n$  where  $A_j$  is the alternating group on  $j$  variables. My preprint [Jau19] describes such an algebra defined below:

**Definition 2.1** ([Jau19]). The *alternating analogue* of  $U(\mathfrak{gl}_n)$ , denoted  $\mathcal{A}(\mathfrak{gl}_n)$ , is defined as the subalgebra of  $(\mathbb{C}\langle x_{ki} \mid 1 \leq i \leq k \leq n \rangle \#\langle \delta^{\ell j} \mid 1 \leq j \leq \ell \leq n-1 \rangle_{\text{grp}})^{\mathbb{A}_n}$  generated by  $U_n \cup \{\mathcal{V}_2, \mathcal{V}_3, \dots, \mathcal{V}_n\}$ . Explicitly,  $\mathcal{A}(\mathfrak{gl}_n)$  is the subalgebra generated by

$$\begin{aligned} X_k^\pm &= \sum_{i=1}^k (\delta^{ki})^{\pm 1} a_{ki}^\pm && \text{for } k = 1, \dots, n-1, \\ X_{kk} &= \sum_{j=1}^k (x_{kj} + j - 1) - \sum_{i=1}^{k-1} (x_{k-1,i} + i - 1) && \text{for } k = 1, \dots, n, \\ \mathcal{V}_k &= \mathcal{V}_k(x_{k1}, \dots, x_{kk}) = \prod_{i < j} (x_{ki} - x_{kj}) && \text{for } k = 1, \dots, n-1, \end{aligned}$$

where  $a_{ki}^\pm$  are described by Futorny and Ovsienko in [FO10].

This provides the first natural example of a Galois ring whose ring  $\Gamma$  is not a semi-Laurent polynomial ring, that is, a tensor product of polynomial rings and Laurent polynomial rings. Additionally, our symmetry group  $A_1 \times A_2 \times \cdots \times A_n$  is not a complex reflection group while the research in this area has been almost exclusively focused on complex reflection groups. The algebra  $\mathcal{A}(\mathfrak{gl}_n)$  is an extension of  $U(\mathfrak{gl}_n)$  by  $n - 1$  elements  $\mathcal{V}_2, \dots, \mathcal{V}_n$ . I prove some properties of  $\mathcal{A}(\mathfrak{gl}_n)$  that are quite similar to  $U(\mathfrak{gl}_n)$ :

**Proposition 2.2** ([Jau19], Proposition 2.2).

- (i)  $U(\mathfrak{gl}_n) \cong U_n \subset \mathcal{A}(\mathfrak{gl}_n)$ ,
- (ii)  $\mathcal{A}(\mathfrak{gl}_n)$  is a Galois  $\tilde{\Gamma}$ -ring,
- (iii)  $\mathcal{V}_n$  is central in  $\mathcal{A}(\mathfrak{gl}_n)$ ,
- (iv)  $Z(\mathcal{A}(\mathfrak{gl}_n)) \cong \mathbb{C}[x_1, \dots, x_n]^{A_n}$ ,
- (v) there is a chain of subalgebras  $\mathcal{A}(\mathfrak{gl}_1) \subset \mathcal{A}(\mathfrak{gl}_2) \subset \cdots \subset \mathcal{A}(\mathfrak{gl}_n)$ ,
- (vi)  $\mathcal{A}(\mathfrak{gl}_n)$  is a minimal extension of  $U(\mathfrak{gl}_n)$  with properties (iv) and (v).

Property (iv) shows that the ‘‘Weyl Group’’ of  $\mathcal{A}(\mathfrak{gl}_n)$  is the alternating group  $A_n$ , in the sense that there is a natural extension  $\tilde{\varphi}_{\text{HC}}$  of the Harish-Chandra homomorphism  $\varphi_{\text{HC}}: Z(U(\mathfrak{gl}_n)) \rightarrow S(\mathfrak{h}) \cong \mathbb{C}[x_1, \dots, x_n]$ , such that

$$\tilde{\varphi}_{\text{HC}}: Z(\mathcal{A}(\mathfrak{gl}_n)) \xrightarrow{\cong} \mathbb{C}[x_1, \dots, x_n]^{A_n}.$$

Additionally, I provide a technique to turn a general Galois ring into a Galois order that is related to localization (see Theorem 6.2 in [Jau19]). The thought of changing a Galois ring into a Galois order was something that had not been considered in the field before. This result provides some connections between three different principal Galois orders for a general  $(\Lambda, G, \mathcal{M})$ . There is the following chain of embeddings:

$$(\Lambda \# \mathcal{M})^G \hookrightarrow \mathcal{K}_\Gamma \hookrightarrow (\text{Frac}(\Lambda) \# \mathcal{M})^G.$$

Hartwig showed that  $(\Lambda \# \mathcal{M})^G$  is always a principal Galois  $\Gamma$ -order ([Har20] Lemma 2.10 and Lemma 2.16), he defined the *standard Galois  $\Gamma$ -order* denoted  $\mathcal{K}_\Gamma$  ([Har20] Definition 2.22 and Theorem 2.21) which contains all principal Galois  $\Gamma$ -orders, and he showed that  $(\text{Frac}(\Lambda) \# \mathcal{M})^G$  is a (principal) Galois  $\Gamma$ -order iff  $\Lambda$  is a field (i.e.  $\Lambda = \text{Frac}(\Lambda)$ ). My technique in a sense brings the left and right ends closer together. Even if you cannot directly apply the technique, it will allow you to describe a new Galois order related to your original Galois ring to describe Gelfand-Tsetlin modules over the original Galois ring.

Finally, I compute the division ring of fractions and prove, for  $n \leq 5$ ,  $\mathcal{A}(\mathfrak{gl}_n)$  satisfies the Gelfand-Kirillov conjecture (see [GK66]). For the latter, I use Maeda’s positive solution to Noether’s problem for the alternating group  $A_5$  [Mae89], and Futorny-Schwarz’s Theorem 1.1 in [FS18b].

## 3 Ongoing work

### 3.1 Alternating nil Hecke algebras

I am currently working on two different projects. The first of these involves describing a complete presentation for the *standard flag order* (see Definition 2.2 in [Web19]) in the setting of  $U(\mathfrak{gl}_n)$ , complex reflection groups, and the alternating group algebra. In [Mit01], there is a description of a  $q$ -analogue of the alternating group, the alternating Hecke algebra, as a subalgebra of the Iwahori-Hecke algebra of the symmetric group. The current work in the field has demonstrated in this setting that the standard flag order associated to the data  $(\mathbb{C}[x_1, \dots, x_n], S_n, 1)$  is isomorphic to the nil Hecke algebra (see [GKV97] and [Rou05]). In my dissertation, I proved that the standard order for the data  $(\mathbb{C}[x_1, \dots, x_n], A_n, 1)$  is a subalgebra of this nil Hecke algebra. I am currently working on using the alternating Hecke algebra of Mitsunashi from [Mit01] to describe this subalgebra that I am calling the *nil Hecke algebra of the alternating group*. Additionally, I have the following conjecture related to this that I am currently trying to prove:

**Conjecture 1.** *Let  $(\Lambda_i, \mathcal{M}_i, W_i)$  for  $i = 1, 2$  be the datum for standard flag orders  $\mathcal{F}_{\Lambda_i}$ . Let  $\Lambda = \Lambda_1 \otimes \Lambda_2$ ,  $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$ , and  $W = W_1 \times W_2$ . The standard flag order for the datum  $(\Lambda, \mathcal{M}, W)$  is*

$$\mathcal{F}_{\Lambda} = \mathcal{F}_{\Lambda_1} \otimes \mathcal{F}_{\Lambda_2}.$$

I have proved an embedding  $\mathcal{F}_{\Lambda} \hookrightarrow \mathcal{F}_{\Lambda_1} \otimes \mathcal{F}_{\Lambda_2}$  in my dissertation, and I am working towards showing it is surjective.

### 3.2 The orthogonal Lie algebra

The second project is related to describing Galois orders in cases other than type  $A$ . To this end, I am working with Jonas Hartwig to construct formula free descriptions of the Galois order realization of  $U(\mathfrak{gl}_n)$  following the techniques in [GKV97]. This should allow for us to avoid the difficulties with the Gelfand-Tsetlin formulas for the other types. As such, we currently have the following conjecture for types  $B/D$ .

**Conjecture 2.** *The algebra  $U(\mathfrak{so}_n)$  is realizable as a Galois order over its Gelfand-Tsetlin subalgebra.*

Some work by Colarusso and Evens (see [CE18]) provides some evidence that the type  $B/D$  scenario is similar enough to type  $A$ . Both of these would greatly further the study of Gelfand-Tsetlin modules.

## 4 Future work

In my preprint [Jau19], I have conjectured a presentation for  $\mathcal{A}(\mathfrak{gl}_3)$  and I plan to show that it is a presentation in the near future. I have some thoughts on describing some infinite-dimensional weight modules for  $\mathcal{A}(\mathfrak{gl}_n)$ . In [Mat00], it is shown that any irreducible torsion-free infinite-dimensional weight module with finite-dimensional weight spaces is isomorphic to a “twisted localization” of  $L(\lambda)$ , the unique irreducible quotient of the Verma module associated to the weight  $\lambda$ . I hope to

relate these types of modules for  $\mathcal{A}(\mathfrak{gl}_n)$  to  $U(\mathfrak{gl}_n)$ -modules in the same way as I did for finite-dimensional modules (see Theorem 5.2 in [Jau19]). Some additional directions for  $\mathcal{A}(\mathfrak{gl}_n)$  involve connecting it to the theory of Lie superalgebras, and Schur-Weyl duality for the alternating group.

In the future, I would also like to obtain a more natural description of the properties of Galois orders related to algebraic geometry. Many properties that may seem perplexing when viewed through the lens of algebra, could be better understood geometrically.

Additionally, a problem that I am fascinated by is finding a Galois order realization of  $U(\mathfrak{sp}_{2n})$ , doing so would prove the Gelfand-Kirillov Conjecture for  $\mathfrak{sp}_{2n}$  which has been open for over 50 years.

## 5 Undergraduate work

As I stated above, most of the work in this area has been focused in the setting where  $G$  is a complex reflection group. This leaves several accessible problems for small groups that are not complex reflection groups. For example, the following two problems could easily be studied by strong undergraduates:

**Open Problem 1.** *Describe the standard flag order in the setting  $\Lambda = \mathbb{C}[x_1, x_2]$ ,  $\mathcal{M} = \langle \delta_1, \delta_2 \rangle_{grp}$  where  $\delta_i(x_j) = x_j - \delta_{ij}$ , and  $G = Q_8$  the quaternion group acting in the natural way on  $\Lambda$ .*

**Open Problem 2.** *Describe the standard flag order in the setting  $\Lambda = \mathbb{C}[x_1, x_2, x_3, x_4]$ ,  $\mathcal{M} = \langle \delta_1, \delta_2, \delta_3, \delta_4 \rangle_{grp}$  where  $\delta_i(x_j) = x_j - \delta_{ij}$ , and  $G = D_4$  the dihedral group of order 8 acting in the natural way on  $\Lambda$ .*

While the dihedral group  $D_n$  of order  $2n$  for  $n \geq 4$  can be realized as a complex reflection group when acting on 2 dimensional vector spaces, it is not generated by reflections when action on  $n$  dimensional space.

These would provide new and unique examples of flag orders in a field that has been dominated by complex reflection groups, and would be analogues to the nilHecke algebra of the symmetric group

## References

- [Bav92] V. V. Bavula. “Generalized Weyl algebras and their representations.” English. In: *St. Petersburg. Math. J.* 4.1 (1992), pp. 71–92. ISSN: 1061-0022; 1547-7371/e.
- [CE18] Mark Colarusso and Sam Evens. “The Complex Orthogonal Gelfand-Zeitlin System”. In: *arXiv e-prints*, arXiv:1808.04424 (Aug. 2018), arXiv:1808.04424. arXiv: 1808.04424 [math.RT].
- [DFO94] Y.A. Drozd, V.M. Futorny, and S.A. Ovsienko. “Harish-Chandra Subalgebras and Gelfand-Zetlin Modules”. In: *Finite Dimensional Algebras and Related Topics* 424 (1994). DOI: [https://doi.org/10.1007/978-94-017-1556-0\\_5](https://doi.org/10.1007/978-94-017-1556-0_5).
- [FS18a] V. Futorny and J. Schwarz. “Quantum Linear Galois Algebras”. In: *arXiv e-prints* (Apr. 2018). arXiv: 1804.08120 [math.RT].

- [FH14] Vyacheslav Futorny and Jonas Hartwig. “Solution of a  $q$ -difference Noether problem and the quantum Gelfand–Kirillov conjecture for  $\mathfrak{gl}_N$ ”. In: *Mathematische Zeitschrift* 276 (Feb. 2014). DOI: 10.1007/s00209-013-1184-3.
- [FMO10] Vyacheslav Futorny, Alexander Molev, and Serge Ovsienko. “The Gelfand–Kirillov conjecture and Gelfand–Tsetlin modules for finite  $W$ -algebras”. In: *Advances in Mathematics* 223.3 (2010), pp. 773–796. ISSN: 0001-8708. DOI: <https://doi.org/10.1016/j.aim.2009.08.018>. URL: <http://www.sciencedirect.com/science/article/pii/S0001870809002655>.
- [FO10] Vyacheslav Futorny and Serge Ovsienko. “Galois orders in skew monoid rings”. In: *Journal of Algebra* 324.4 (2010), pp. 598–630. ISSN: 0021-8693. DOI: <https://doi.org/10.1016/j.jalgebra.2010.05.006>. URL: <http://www.sciencedirect.com/science/article/pii/S0021869310002267>.
- [FO14] Vyacheslav Futorny and Serge Ovsienko. “Fibers of characters in Gelfand–Tsetlin categories”. In: *Transactions of the American Mathematical Society* 366 (Aug. 2014). DOI: 10.1090/S0002-9947-2014-05938-2.
- [FS18b] Vyacheslav Futorny and João Schwarz. “Noncommutative Noether’s Problem vs Classical Noether’s Problem”. In: *arXiv e-prints* (May 2018). arXiv: 1805.01809 [math.RA].
- [Fut+18] Vyacheslav Futorny et al. “Gelfand–Tsetlin Theory for Rational Galois Algebras”. In: *arXiv e-prints* (Jan. 2018). arXiv: 1801.09316 [math.RT].
- [GT50] Israel M. Gelfand and Michael L. Tsetlin. “Finite-dimensional representations of the group of unimodular matrices”. In: *Dokl. Akad. Nauk Ser. Fiz.* 71 (1950), pp. 825–828.
- [GK66] Izrail M. Gelfand and Alexander A. Kirillov. “Sur les corps liés aux algèbres enveloppantes des algèbres de Lie”. fr. In: *Publications Mathématiques de l’IHÉS* 31 (1966), pp. 5–19. URL: [http://www.numdam.org/item/PMIHES\\_1966\\_\\_31\\_\\_5\\_0](http://www.numdam.org/item/PMIHES_1966__31__5_0).
- [GKV97] Victor Ginzburg, Mikhail Kapranov, and Eric Vasserot. “Residue Construction of Hecke Algebras”. In: *Advances in Mathematics* 128.1 (1997), pp. 1–19. ISSN: 0001-8708. DOI: <https://doi.org/10.1006/aima.1997.1620>. URL: <http://www.sciencedirect.com/science/article/pii/S0001870897916200>.
- [Har20] Jonas T. Hartwig. “Principal Galois orders and Gelfand–Zeitlin modules”. In: *Advances in Mathematics* 359 (2020), p. 106806. ISSN: 0001-8708. DOI: <https://doi.org/10.1016/j.aim.2019.106806>. URL: <http://www.sciencedirect.com/science/article/pii/S0001870819304232>.
- [Jau19] Erich C. Jauch. “An Alternating Analogue of  $U(\mathfrak{gl}_n)$  and Its Representations”. In: *arXiv e-prints* (July 2019). arXiv: 1907.13254 [math.RT].
- [LW19] Elise LePage and Ben Webster. *Rational Cherednik algebras of  $G(\ell, p, n)$  from the Coulomb perspective*. 2019. arXiv: 1912.00046 [math.RA].
- [LM73] J. Lepowsky and G. W. McCollum. “On the determination of irreducible modules by restriction to a subalgebra”. In: *Trans. Amer. Math. Soc.* 176 (1973), pp. 45–57. ISSN: 0002-9947. DOI: 10.2307/1996195. URL: <https://doi.org/10.2307/1996195>.

- [Mae89] Takashi Maeda. “Noether’s problem for  $A_5$ ”. In: *Journal of Algebra* 125.2 (1989), pp. 418–430. ISSN: 0021-8693. DOI: [https://doi.org/10.1016/0021-8693\(89\)90174-9](https://doi.org/10.1016/0021-8693(89)90174-9). URL: <http://www.sciencedirect.com/science/article/pii/0021869389901749>.
- [Mat00] Olivier Mathieu. “Classification of irreducible weight modules”. In: *Annales de l’Institut Fourier* 50 (Jan. 2000). DOI: 10.5802/aif.1765.
- [MV18] Volodymyr Mazorchuk and Elizaveta Vishnyakova. “Harish-Chandra modules over invariant subalgebras in a skew-group ring”. In: *arXiv e-prints* (Nov. 2018). arXiv: 1811.00332 [math.RT].
- [Mit01] Hideo Mitsuhashi. “The  $q$ -Analogue of the Alternating Group and Its Representations”. In: *Journal of Algebra* 240.2 (2001), pp. 535–558. ISSN: 0021-8693. DOI: <https://doi.org/10.1006/jabr.2000.8715>. URL: <http://www.sciencedirect.com/science/article/pii/S0021869300987155>.
- [Ovs03] Serge Ovsienko. “Strongly nilpotent matrices and Gelfand–Zetlin modules”. In: *Linear Algebra and Its Applications - LINEAR ALGEBRA APPL* 365 (May 2003), pp. 349–367. DOI: 10.1016/S0024-3795(02)00675-4.
- [Ros95] Alexander L. Rosenberg. *Noncommutative algebraic geometry and representations of quantized algebras*. Vol. 330. Mathematics and its Applications. Kluwer Academic Publishers Group, Dordrecht, 1995, pp. xii+315. ISBN: 0-7923-3575-9. DOI: 10.1007/978-94-015-8430-2. URL: <https://doi.org/10.1007/978-94-015-8430-2>.
- [Rou05] Raphael Rouquier. *Representations of rational Cherednik algebras*. 2005. arXiv: math/0504600 [math.RT].
- [Web19] Ben Webster. “Gelfand-Tsetlin modules in the Coulomb context”. In: *arXiv e-prints* (Apr. 2019). arXiv: 1904.05415 [math.RT].
- [Žel73] D. P. Želobenko. *Compact Lie groups and their representations*. Translated from the Russian by Israel Program for Scientific Translations, Translations of Mathematical Monographs, Vol. 40. American Mathematical Society, Providence, R.I., 1973, pp. viii+448.